CSE 20 Week 10 Discussion Worksheet

Problem 1.

Let $f : A \to B$ and $g : B \to C$ be functions. Then the *composition of* g *with* f, denoted by $g \circ f$, is the function

$$g \circ f : A \to C$$

such that for every input $a \in A$,

$$(g \circ f)(a) = g(f(a)).$$

That is, we first apply f to a, and then apply g to the result. Convince yourself that $g \circ f$ is well-defined.

(a) Show that if both f and g are one-to-one functions, then $g \circ f$ is also one-to-one.

(b) Show that if both f and g are onto functions, then $g \circ f$ is also onto.

Problem 2.

Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

(a) a is taller than b.

(b) a and b were born on the same day.

(c) a and b have a common class this quarter.

Problem 3.

Let A be a finite set with n elements, and let B be a finite set with m elements. How many distinct relations are there from A to B?

Problem 4.

Let $S = \mathbb{Z} \times (\mathbb{Z} \setminus \{0\})$. Consider the following relation on S defined as

$$R = \{((a, b), (c, d)) \in S \times S \mid ad = bc\}.$$

(a) Write down a couple of elements in the relation R to get a flavor of it.

(b) Show that R is an equivalence relation.

(c) What is the equivalence class $[(2, 3)]_R$ with respect to this relation?

(d) What familiar mathematical set do the equivalence classes of this relation correspond to?