CSE 20: Discrete Mathematics
Week 8 Discussion

Question 1. (Rosen p.350) Let $P(n)$ be the statement that, for the positive integer n

$$1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$$

(a) What is the statement $P(1)$?

(b) Show that $P(1)$ is true, completing the basis step of a proof that $P(n)$ is true for all positive integers n.

(c) What is the inductive hypothesis of a proof that $P(n)$ is true for all positive integers n?

(d) What do you need to prove in the inductive step of a proof that $P(n)$ is true for all positive integers n?

(e) Complete the inductive step of a proof that $P(n)$ is true for all positive integers n, identifying where you use the inductive hypothesis.

(f) Explain why these steps show that this formula is true whenever n is a positive integer.

Question 2. (Rosen p.350)

(a) Find a formula for

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

by examining the values of this expression for small values of n.

(b) Prove the formula you conjectured in part (a).

Question 3. (Rosen p.95) Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

Question 4. (Rosen p.95) Prove that if n is an integer and $3n + 2$ is even, then n is even using

(a) a proof by contraposition.

(b) a proof by contradiction

Question 5. (Rosen p.95) Prove that if m and n are integers and mn is even, then m is even or n is even.

Question 6. (CSE20FA22) Consider the Fibonacci sequence as a function $F : \mathbb{N} \to \mathbb{N}$

$$F(0) = 0, \quad F(1) = 1$$

$$F(n) = F(n-1) + F(n-2)$$

Prove that for any natural number $k \geq 11$, $F(k) \geq 1.5^k$. Hint: Use $k = 11, 12$ as base cases, then proceed with strong induction.