## **Division algorithm**

**Integer division and remainders** (aka The Division Algorithm) Let n be an integer and d a positive integer. There are unique integers q and r, with  $0 \le r < d$ , such that n = dq + r. In this case, d is called the divisor, n is called the dividend, q is called the quotient, and r is called the remainder.

Because these numbers are guaranteed to exist, the following functions are well-defined:

- **div** :  $\mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$  given by **div** ((n, d)) is the quotient when n is the dividend and d is the divisor.
- mod :  $\mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Z}$  given by mod ( (n, d) ) is the remainder when n is the dividend and d is the divisor.

Because these functions are so important, we sometimes use the notation  $n \operatorname{div} d = \operatorname{div} ((n, d))$  and  $n \operatorname{mod} d = \operatorname{mod} ((n, d))$ .

**Pro-tip**: The functions div and mod are similar to (but not exactly the same as) the operators / and % in Java and python.

Example calculations:

 $20~{\rm div}~4$ 

 $20 \mod 4$ 

 $20~{\rm div}~3$ 

 $20 \mod 3$ 

-20 div 3

 $-20 \mod 3$