

Logical equivalence identities

(Some) logical equivalences

Can replace p and q with any compound proposition

$$\neg(\neg p) \equiv p$$

Double negation

$$p \vee q \equiv q \vee p$$

$$p \wedge q \equiv q \wedge p$$

Commutativity Ordering of terms

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Associativity Grouping of terms

$$p \wedge F \equiv F \quad p \vee T \equiv T \quad p \wedge T \equiv p \quad p \vee F \equiv p$$

Domination aka short circuit evaluation

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

DeMorgan's Laws

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Contrapositive

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$\neg(p \leftrightarrow q) \equiv p \oplus q$$

$$p \leftrightarrow q \equiv q \leftrightarrow p$$

Extra examples:




$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because _____

$p \rightarrow q$ is not logically equivalent to $q \rightarrow p$ because _____


Logical operators truth tables

Truth tables: Input-output tables where we use T for 1 and F for 0.

Input		Output		
p	q	Conjunction $p \wedge q$	Exclusive or $p \oplus q$	Disjunction $p \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

		
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Input	Output
p	Negation $\neg p$
T	F
F	T



Logical equivalence

Logical equivalence : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T .

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F .

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Logical equivalence extra example

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					